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Performance of a Diffuse Convolution Code on HF Error Statistics

WAYNE M. JEWETT

Systems Integration and Instrumentation Branch
Communications Sciences Division

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Analysis of the performance of a simulated decoder for the Kohlenberg and Forney one-half-rate, threshold-decodable, diffuse convolutional code, on error data obtained on a simulated HF radio channel, indicates that background errors may be significantly suppressed in the presence of burst errors by using a diffusion factor that introduces a total coding delay of about 100 ms. The precise value of the diffusion factor must be tailored to the modem frame size. Furthermore, when the modulation technique introduces correlation between errors on bit pairs, it is preferable to confine the errors to corresponding information and parity bits. Random
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20. ABSTRACT (Continued)

transmission errors on the information bits are shown to affect the decoder more than errors on the parity bits.

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PERFORMANCE OF A DIFFUSE CONVOLUTIONAL CODE ON
HF ERROR STATISTICS

INTRODUCTION

The Naval Research Laboratory (NRL) is engaged in the design of a digital voice system for high-frequency (HF) radio circuits. This system will incorporate a linear predictive encoder (LPC) as the digital voice terminal. Use of forward acting error correction in the HF modem to protect a portion of the LPC data is being considered.

Use of a short block code to protect a portion of the LPC data was considered in the FRANKLIN study [1] and in an NRL memorandum [2]. The use of a threshold-decodable, diffuse convolutional code is considered in this report. This code is simple to implement and provides flexibility in the proportion of the LPC data that may be encoded.

Test have been run using diffuse convolutional codes on HF channels [3-5]. Long time spreads in the codes tested were chosen to combat wideband channel burst noise. Results were favorable for the use of these codes in a data transmission system, but large time delays are undesirable in digital voice applications.

Improving the background error rate by the use of short time spreads (diffusion factors) is the object of this work. The relationship between the diffusion factor and the modem modulation format is investigated, and the sensitivity of the decoded error rate to different error rates on the information and parity bits is examined. No attempt is made to encode over the burst errors produced by wideband fading.

HF CHANNEL ERRORS

Bit errors incurred in the transmission of digital data over an HF channel may be described in terms of four classes:

1. Random errors due to low signal-to-noise conditions

2. Periodic errors caused by

   a. Effects of narrowband frequency-selective fading on a frequency-division multiplex (FDM) modem.
   b. Effects of narrowband interference on a FDM modem.
   c. Poor filter characteristics in the radio equipment that discriminates against the end channels in a FDM modem.
   d. Error propagation between signaling elements (bauds) due to the modulation technique, such as differentially coherent phase shift keying (DPSK).

3. Consecutive errors attributed to

   (a) Frequency-selective fading on adjacent channels of a FDM modem, producing multiple errors in a given baud.
   (b) Use of a nonbinary modulation technique, such as four-phase DPSK.

4. Error bursts that may be due to wideband fading or wideband interference. The duration of the burst is controlled by the input signal and the nonlinear characteristics of the receiving equipment.

A communication channel is often characterized in terms of its average bit error rate (BER). This is a useful term for the comparison of various coding parameters when the undecoded bit errors are randomly distributed. It provides a poor indication of the performance of a decoder for a compound channel composed of burst errors superimposed on a background of random and periodic errors; the undecoded (raw) BER could be dominated by the bursts. Under these conditions it is useful to characterize errors in terms of error-free blocks or the cumulative distribution of gaps. A gap is the number of error-free bits between two errors. These statistics provide information about the clustering of the errors and the background error level. These are useful channel statistics for accessing the acceptability of a digital voice circuit. Error-free blocks and gap distributions are used in this study to characterize the performance obtained with a diffuse convolutional code.

SIMULATED HF CHANNEL

The performance of the convolutional code was determined using errors obtained from a 2400-bit/s, 16-tone, four-phase DPSK modem and a simulated HF channel. The modem was a software implementation of the MIL-STANDARD-188 HF modem. It was programed to operate in real time on the Sylvania Programmable Signal Processor (PSP) [6]. The real-time HF channel simulator was also a software implementation. It was programed on the CSP-30 digital computer. The HF channel model is a tapped delay line representation of the time-varying channel, based on the work of Richman and Monsen [7]. General characteristics of the simulator are reported in Ref. 8. The path conditions for the test channel are given in Table 1. These conditions correspond to the worst simulated channel conditions used in the recent consortium digital-voice terminal tests [9]. An average BER of 1.9×10^{-2} was measured from the transmission of 2 million bits. The detected errors for a 2047-bit psuedorandom sequence were recorded on digital tape to provide the data base for evaluating the diffuse convolutional code.

CONVOLUTIONAL DECODER

A threshold-decodable, diffuse convolutional decoder was programed to operate in nonreal time on a PDP-11/45 computer. The code selected was a variation on the work of Massey [10] and is described by Kohlenberg and Forney [11].

Table 1 — Path Parameters for Simulated HF Channel

Path	Delay (ms)	Relative Amplitude	Mean Frequency Offset (Hz)	Doppler Spread (Hz)
1	0	1.0	0.1	0.1
2	0.7	0.7	0.2	0.5
3	1.5	0.5	0.5	1.0
4	2.2	0.25	1.0	2.0

The generator polynomial for the encoder is

$$g(D) = 1 + D^x + D^{2x} + D^{3x+1}$$

where x is the diffusion factor. This is a double error correction, one-half rate, systematic code using a feedback decoder. A block diagram of the decoder is shown in Fig. 1. The performance of the software decoder was verified using a simulated binary symmetric channel with a crossover error probability of p. For this condition the predicted decoded error rate [11] is $166\,p^3$. The results for the random error tests are shown in Fig. 2.
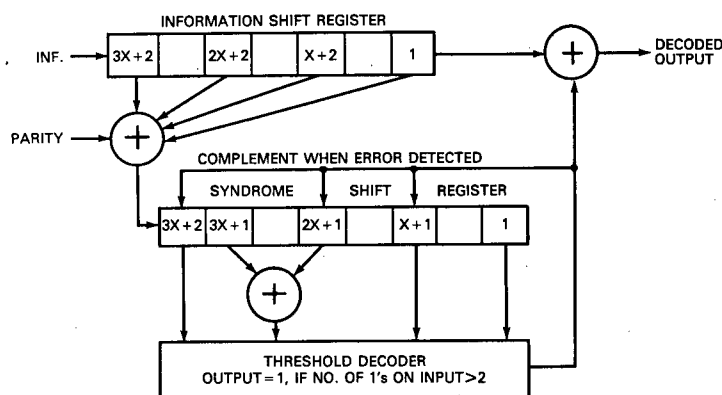

Fig. 1 — One-half rate diffuse decoder

This decoder is an attractive candidate for incorporation in the HF digital voice modem for the following reasons.

1. Tests on HF channels [3-5] indicated that this code with a large diffusion factor performed as well as or better than short block codes with interleaving.
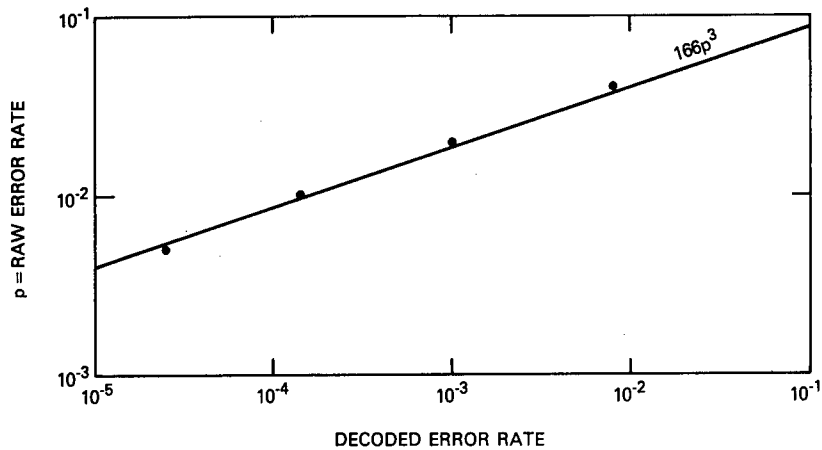
Fig. 2 — Performance of one-half rate diffuse decoder with
random errors

2. The decoder is simple to implement in either hardware or software; thus, it would place a minimum burden on a processor programed for the digital voice, coding and modulation functions.

3. The use of a convolutional code rather than a block code provides greater flexibility in selecting the number of bits to be encoded.

4. Selection of a systematic code over a nonsystematic code permits the decoding process to be bypassed if necessary.

5. This convolutional code is being considered for the tactical switched communication system [12], where for many conditions it is the preferred error control technique.

TESTS RESULTS

Initial tests using recorded error data determined the effects of the diffusion factor on the decoded error rate. The diffusion factor was varied from 1 to 50 bits. This represented a maximum decoding delay of 152 bits, or 126 ms. Simulation results are shown in Fig. 3 in terms of BER vs diffusion factor. The information and parity bits were transmitted with no delay, as one 2-bit symbol.

The data in Fig. 3 indicate the sensitivity of the decoder to the value of the diffusion factor. This variation in decoder performance is basically the result of periodic errors in the input data; that is, errors that occur at a multiple of the modem frame rate. Decoder performance is optimized when the diffusion factor is not related to the modem frame period. This is similar to the results reported for block codes with interleaving [4,13].

The 16-tone modem produced periodic errors at multiples of 16 bits in both the information and parity bit streams. Best decoder performance is obtained when no tap spacing in the encoder is a multiple of 16. The difference values between the taps for this encoder are
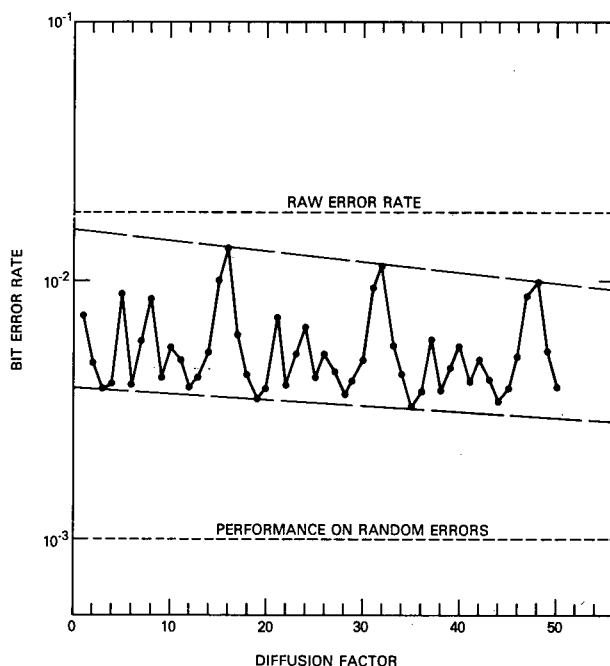
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Fig. 3 — Decoder performance vs diffusion factor on HF errors, for simultaneous transmission of information and parity bits on common tone of a 16-tone, four-phase DPSK modem

shown in Table 2 in terms of the diffusion factor. Table 3 relates each of these difference values to a diffusion factor that would produce poor decoder performance. A comparison may be made among the diffusion factors where poor performance was expected, as shown in Table 3, and the measured performance as shown in Fig. 3.

Table 2 — Spacing Between Taps in Diffuse
Convolutional Encoder

Values	1	$x + 2$	$2x + 2$	$3x + 2$
1	0	$x + 1$	$2x + 1$	$3x + 1$
$x + 2$		0	x	$2x$
$2x + 2$			0	x
$3x + 2$				0

The measured data confirm the analysis results, which indicate that selection of the diffusion factor should be based on knowledge about periodic errors in the input data. Best performance was obtained for diffusion factors equal to three plus an integer multiple of the number of information bits in the modem frame (d min = 16M + 3; $\bar{M}$ = 0, 1, 2). The improvement in terms of BER for increasing values of M was not significant for the small values considered.

Table 3 — Diffusion Factors Where Poor Performance Would Be Expected
due to Periodic Errors at the Modem Frame Rate

Spacing Between Taps	Diffusion Value for Which Tap Spacing is a Multiple of 16						
	16	32	48	64	80	96	112
x	16	32	48	64	80	96	112
$x + 1$	15	31	47	63	79	95	111
$2x$	8	16	24	36	40	48	56
$2x + 1$	7.5	15.5	23.5	31.5	39.5	47.5	55.5
$3x + 1$	5	10.3	15.6	21	26.3	31.6	37

Figures 4 and 5 show, on a logrithmitic probability scale, the cumulative distribution of gaps. The cases shown are for no decoding and for decoding with diffusion factors of 3, 19, and 35. These factors correspond to the points at which minimum BERs were obtained. For the no-decoding case, the presence of a large number of gaps at multiples of the modem frame size is evidenced by the abrupt steps in the data. These are the results of periodic errors occurring at the modem frame rate. These periodic errors are largely corrected by the code, but a new source of periodic errors related to the diffusion factor is incurred. This results from error propagation in the decoder.
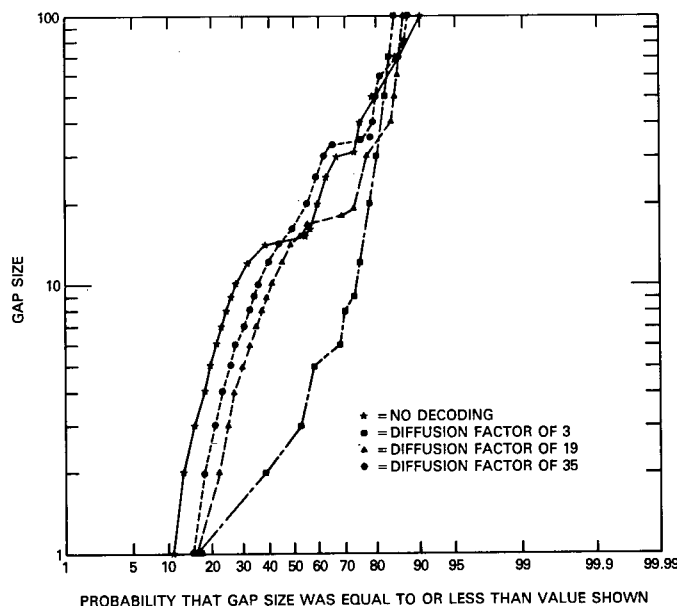

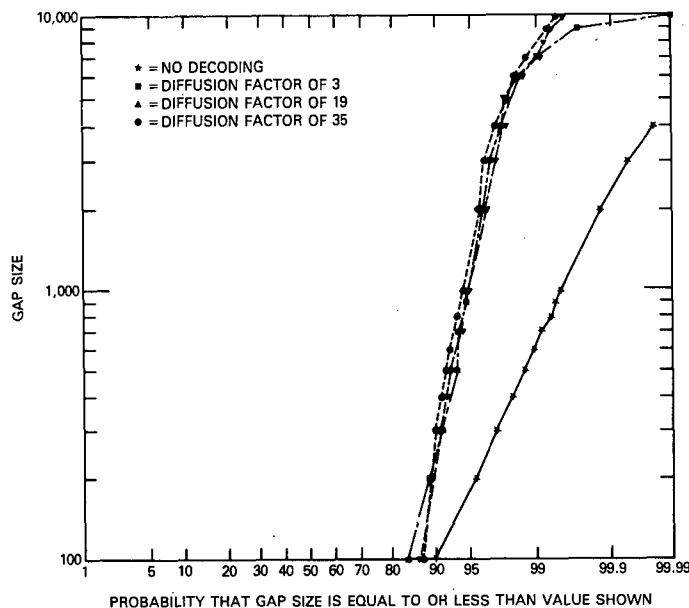Fig. 4 — Cumulative distribution of gap sizes for gaps of
100 bits or less

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Fig. 5 — Cumulative distribution of gap sizes for gaps of
10,000 bits or less

The convolutional code did not improve the gap distribution for gaps of 100 bits or less. The predominant factor contributing to short gaps is the high density of errors in the large error bursts. The decoder tended to increase the density of the errors in these bursts. This is indicated by a shifting of the data points to the right for the decoded data compared to the undecoded data. This was most evident for short diffusion factors. The distribution of gap size as a function of diffusion factor is shown in Fig. 6. The probability of small gaps was significantly increased for a diffusion factor of three. However, for a factor of 35, these statistics approach those of the undecoded data.

The benefit from coding is very evident as the gap size increased above 100 bits. For example, with no decoding, 95% of the gaps were equal to or less than 180 bits. This corresponds to a maximum error-free period of 180/1200 = 150 ms. In contrast, for a diffusion factor of 35, 95% of the gaps in the decoded data were equal to or less than 1200 bits. This represents a maximum error-free period of 1.0 s. These data indicate that the decoder is reducing the background error rate with little effect on the major error bursts.

The probability of a given number of errors in a modem frame period was the third statistic used to characterize the error patterns. Results are shown in Fig. 7. The decrease in the background errors from the use of the code is evident in the increase in the probability of an error-free frame. This probability was increased from 76.3% for the undecoded case to 96.3% for decoding with a diffusion factor of 35. In addition, the probability of either one, two, or three errors per frame was significantly reduced. The probability of having four or more errors in one frame was approximately the same for both the undecoded and decoded cases.
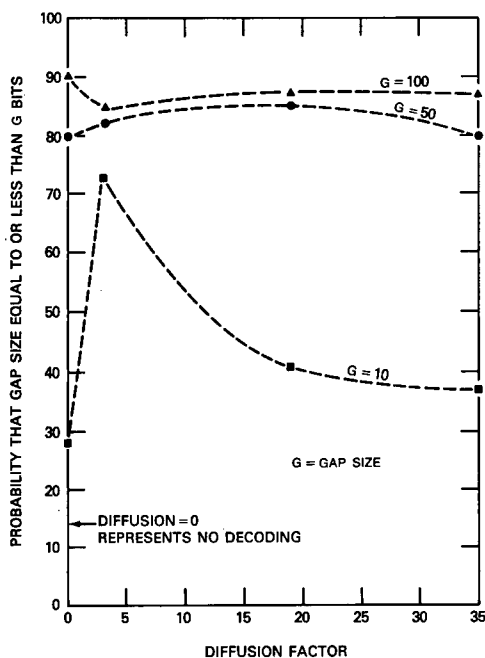
Fig. 6 — Probability that the gap size is equal to or less than a given value, as a function of the diffusion factor.
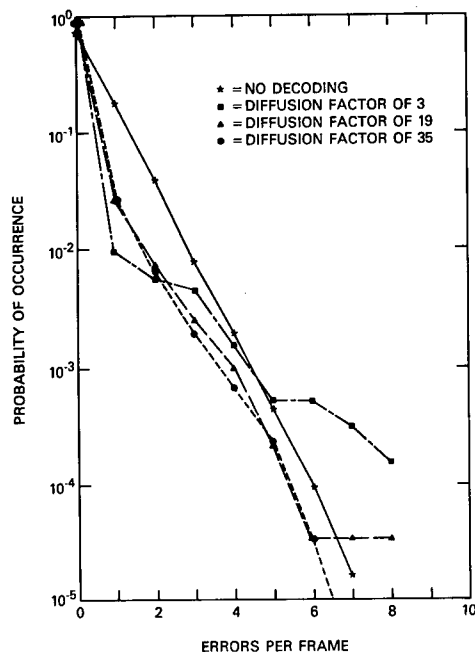

Fig. 7 — Probability of N errors per modem frame

The previous results were obtained for the condition that each information bit and corresponding parity bit are transmitted as one 2-bit symbol on the same tone of the FDM modem. For this condition there was significant correlation between the errors on the information bits and the errors on the parity bits. The extent to which such a correlation existed is shown in Fig. 8. These data show that for an information bit in error, there is a 0.21 probability that the parity bit transmitted on the same tone is also in error. This relationship was reduced significantly between two bits on different tones in the FDM modem.

Test were made to determine whether the decoder performance improves when the parity bits are transmitted with a delay relative to the corresponding information bits. A delay of 8 bits was used. This corresponds to a frequency separation between the 2 bits of 8 tones. Figure 9 shows the results obtained for diffusion factors of 1 to 22. In general, delaying the transmission of the parity bits by 8 bit periods did not provide any improvement.

It was shown in Ref. 11 that the values of the syndromes depend only on the errors and not on the actual values of the transmitted bits. The equations for the four syndromes used by the threshold decoder were also developed in Ref. 11 for the assumption that there had been no previous errors. These syndrome equations are
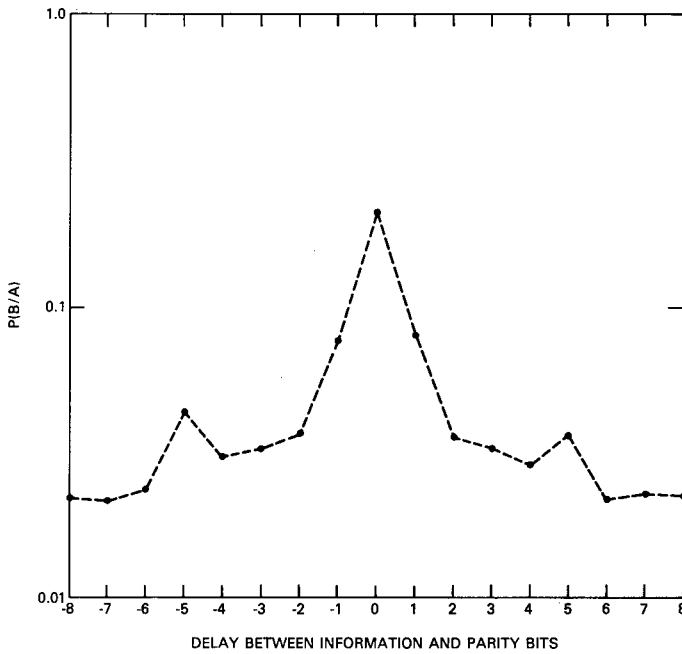
$$S_1 = \left(e^i_1 + e^p_1 \right)$$

(1)

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Fig. 8 — The effect of transmission delay on the conditional probability, P(B/A)
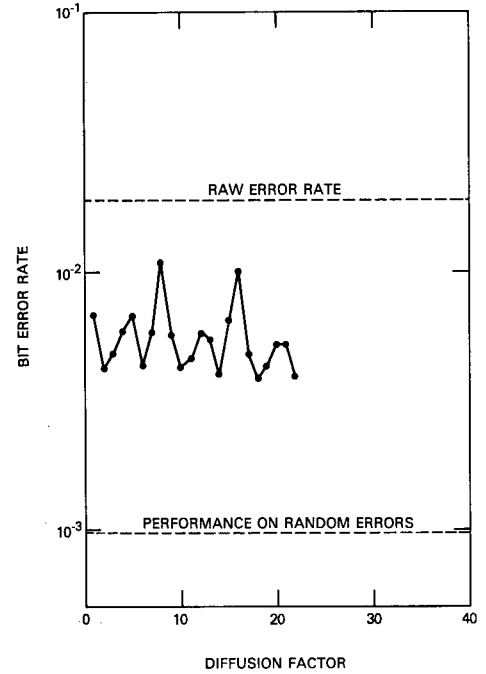
Fig. 9 — Bit error rate vs diffusion factor when parity is delayed eight bit periods

$$S_{x+1} = e_1^i + \left(e_{x+1}^i + e_{x+1}^p\right) \qquad (2)$$

$$S_{2x+1} + S_{3x+1} = e_1^i + e_{2x+1}^p + \left(e_{3x+1}^i + e_{3x+1}^p\right) \qquad (3)$$

$$S_{3x+2} = e_1^i + e_{x+2}^i + e_{2x+2}^i + \left(e_{3x+2}^i + e_{3x+2}^p\right) \quad . \qquad (4)$$

The first bit may be decoded correctly if no more than two errors appear in the 11 bits used in the four equations. The brackets denote that each equation contains a sum of the errors on one information bit and its corresponding parity bit. When there is a high correlation between these errors, there will be a high probability of mutual cancellation. Additional errors may then be corrected. This explains the results showing better performance when each information bit and its parity bit were transmitted on the same tone.

The 11 bits used in the syndrone equations represent the bit to be decoded plus five other information bits and five parity bits. Utilizing five information and five parity bits does not imply that errors on the information bits have the same effect on the decoder as errors on the parity bits. The threshold circuit in the decoder sees only the results of the four syndrome equations. It is these four values that are treated equally. In actuality, the relative effect of errors on the information bits and errors on the parity bits is proportional to the ratio

of the number of information bits to the number of parity bits in each equation. These ratios are 1, 2, 1, and 4 for the four equations, respectively. Thus, syndrome Eq. (2) and (4) would be more sensitive to errors on the information bits then to errors on the parity bits.

It is of interest to extend this analysis to the case where there is correlation between the errors on the information and parity bits. For this condition, Eq. (1) could be canceled and Eq. (2) and (4) could be reduced to the condition in which only the errors on the information bits influence the equation value.

To summarize the above analysis, it is expected that the decoder performance is more sensitive to the errors in the information bits than to errors on the parity bits. In addition, this bias is accentuated when there is some correlation between the errors on an information bit and its parity bit. Additional tests were made to verify this. Tests were made with separately specified error rates for the information and parity bit streams. The results are shown in Fig. 10. These data show that when both error rates are 0.02 the decoded error rate is 0.001. When the BER on the parity bits is held at 0.02 and the BER on the information bits is decreased to 0.01, the decoded BER drops to 0.00035. However, when the reverse test is performed, keeping the BER on the information bits at 0.02 and decreasing the BER on the parity bits to 0.01, the decoded BER dropped to only 0.00055.
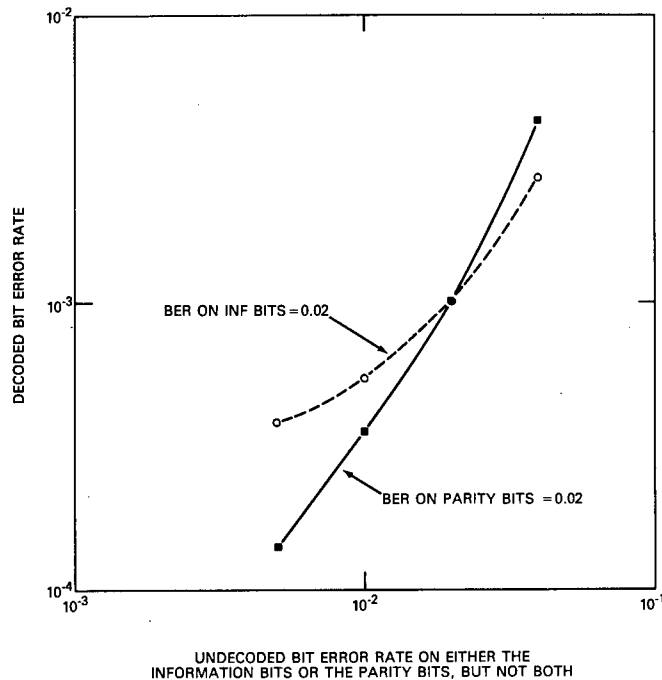

Fig. 10 — Performance of decoder with independently specified random error rates on the information and parity bits

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CONCLUSIONS

The half-rate, threshold-decodable, diffuse convolutional code may be used with small diffusion factors to achieve significant suppression of background errors in the presence of error bursts. The exact amount of diffusion is selected to diminish the effects of periodic errors in the input data. For a 16-tone, four-phase DPSK modem, the best diffusion factor is an integer multiple of 16 plus 3 bits.

Better performance is obtained with a four-phase DPSK modem when each information and parity bit are transmitted without delay on the same tone. This test result is supported with analysis indicating that the decoder may correct more than two errors in a pattern of 11 bits when there is correlation between the errors on the information and parity bits.

Tests with random errors verified the analysis that the decoder is more sensitive to errors on the information bits than to errors on the parity bits.

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